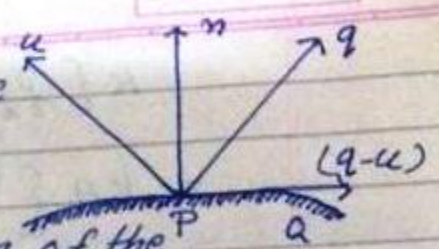
## Boundary Surface:-

$u$ = velocity of surface

$q$ = velocity of fluid




Consider the equation of the Boundary Surface at the Point P

$$F(u, t) = 0$$

Let Q be the velocity of surface at Point P

Consider $n$ is the unit normal vector drawn at the Point P on the Boundary Surface

Since there must be no relative normal velocity at P between the Boundary and the fluid so we must have

$$q \cdot n = u \cdot n$$

$$\Rightarrow (q-u) \cdot n = 0$$

$$\Rightarrow (q-u) \nabla F \qquad (n = \nabla F)$$

At The Point Q

$$F(r+\delta r, t+\delta t) = 0$$

Expanding by Taylor's theorem

$$F(r,t) + \delta r \nabla F + \delta t \cdot \frac{\partial F}{\partial t} = 0$$

$$\delta r \nabla F + \delta r \cdot \frac{\partial F}{\partial t} = 0$$

$$\Rightarrow \frac{\partial F}{\partial t} + \frac{\partial r}{\partial t} \nabla F = 0$$

$$\Rightarrow \frac{\partial F}{\partial t} + u \nabla F = 0$$

As $\partial t \to 0$, $\partial r \to 0$

$$u = \frac{\partial r}{\partial t}$$

$$\Rightarrow \boxed{\frac{\partial F}{\partial t} + q \nabla F = 0}$$ known as eq$^n$ of

$$= K\left\{\frac{\partial}{\partial x}\left(\frac{Ax}{x^2+y^2}\right) + \frac{\partial}{\partial y}\left(\frac{Ay}{x^2+y^2}\right)\right\}$$

$$= KA\left\{\frac{(x^2+y^2)-2x^2}{(x^2+y^2)^2} + \frac{(x^2+y^2)-2y^2}{(x^2+y^2)^2}\right\}$$

Hence the motion is of Potential kind $= 0$

$\Rightarrow$ motion is irrotational

(4) $\Rightarrow$ Velocity Potential exists.

$$q = -\nabla\phi$$

$$\Rightarrow \frac{\partial\phi}{\partial x} = -u = \frac{Au}{x^2+y^2}$$

$$\frac{\partial\phi}{\partial y} = -v = \frac{Ax}{x^2+y^2}$$

$$\frac{\partial\phi}{\partial z} = -w = 0 \Rightarrow \phi \text{ independent of } z$$

$$\boxed{\phi = \phi(x,y)}$$

(5) $\frac{Ay}{x^2+y^2} \Rightarrow \phi = A\tan^{-1}\frac{x}{y} + f(y)$

$\cancel{\frac{Ax}{x^2+y^2} \Rightarrow \phi = \frac{-Ax}{x^2+y^2} = \frac{A}{1+\frac{x^2}{y^2}}(-x-}$

$$\frac{\partial\phi}{\partial y} = f'(y) - \frac{Ax}{x^2+y^2}$$

$$f'(y) = 0$$

$$f(y) = \text{constant}$$

$$\phi(x,y) = A\tan^{-1}\frac{x}{y}$$

c is an absalutly constant.

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$$\frac{\partial F}{\partial t} = \frac{\partial F}{\partial t} + u\frac{\partial F}{\partial x} + v\frac{\partial F}{\partial y} + w\frac{\partial F}{\partial z} = 0 \quad \text{—(1)}$$

Where the velocity component $u, v, w$ are Satisfies the continuity eq$^n$ $\nabla \cdot q = 0$

$$\frac{\partial F}{\partial t} = \frac{2x^2}{a^2}\tan t \sec^2 t - \frac{2y^2}{b^2}\cot t \csc^2 t$$

And $\frac{\partial F}{\partial x} = \frac{2x}{a^2}\tan^2 t$

$$\frac{\partial F}{\partial y} = \frac{2y}{b^2}\cot^2 t$$

From (1) we have

$$\frac{2x^2}{a^2}\tan t \sec^2 t - \frac{2y^2}{b^2}\cot t \csc^2 t + u\frac{2x}{a^2}\tan^2 t + v\frac{2y}{b^2}\cot^2 t = 0$$

$$\Rightarrow \frac{2x}{a^2}\tan t\,(x\sec^2 t + u\tan t) + \frac{2y}{b^2}\cot t\,(-y\csc^2 t + v\cot t) = 0$$

The Boundary condition is

$$x\sec^2 t + u\tan t = 0 \quad \text{—(2)}$$

And $-y\csc^2 t + v\cot t = 0$ —(3)

From (2) and (3)

$$u = -x\sec^2 t \cot t$$

$$u = -(x/\sin t\cos t)$$

and $v = (y/\sin t \cos t)$

which satisfies the equation of continuity

$$\nabla \cdot q = 0$$

$u$, and $v$ with velocity components

Also Normal velocity

$$= \frac{u(\partial F/\partial x) + v(\partial F/\partial y)}{\sqrt{\left(\frac{\partial F}{\partial x}\right)^2 + \left(\frac{\partial F}{\partial y}\right)^2}}$$

Boundary surface $\frac{\partial F}{\partial t} + \frac{\partial r}{\partial t} \nabla F = 0$

$$\Rightarrow \quad \frac{\partial F}{\partial t} + q \nabla F = 0$$

Equation of continuity to a moving Boundary surface

$$\frac{\partial F}{\partial t} + u\frac{\partial F}{\partial x} + v\frac{\partial F}{\partial y} + w\frac{\partial F}{\partial z} = 0$$

If the surface be at rest than $\frac{\partial F}{\partial t} = 0$

$$u\frac{\partial F}{\partial x} + v\frac{\partial F}{\partial y} + w\frac{\partial F}{\partial z} = 0$$

Normal Velocity of the Boundary

$$u \cdot n = \frac{u \cdot \nabla F}{|\nabla F|}$$

$$\boxed{u \cdot n = \frac{\partial F / \partial t}{\sqrt{\left(\frac{\partial F}{\partial x}\right)^2 + \left(\frac{\partial F}{\partial y}\right)^2 + \left(\frac{\partial F}{\partial z}\right)^2}}}$$

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Ex-36/63 Show that $\frac{x^2}{a^2}\tan^2 t + \frac{y^2}{b^2}\cot^2 t = 1$

is a Possible form for the bounding surface of a liquid and find an expression for the normal velocity.

Sol$^n$ The Boundary surface of a liquid is given as

$$F(x, y, z, t) = \frac{x^2}{a^2}\tan^2 t + \frac{y^2}{b^2}\cot^2 t - 1 = 0$$

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$$= \frac{-\frac{x}{\sin t\cos t}\cdot\frac{2x}{a^2}\tan^2 t + \frac{y}{\sin t\cos t}\cdot\frac{2y}{b^2}\cot^2 t}{\sqrt{\left(\frac{2x}{a^2}\tan^2 t\right)^2 + \left(\frac{2y}{b^2}\cot^2 t\right)^2}}$$

$$= \frac{a^2y^2\cot t\,\mathrm{cosec}^2 t + b^2x^2\tan t\sec^2 t}{x^2b^4\tan^4 t + y^2a^4\cot^4 t}$$ Ans.

Ex-13 / 29 Determine the acceleration of a fluid Particle of fixed identity for the velocity field.

$$q = iAx^2y + jBy^2zt + kCzt^2$$

Sol.ⁿ since $q = iu + jv + kw = iAx^2y + jBy^2zt + kCzt^2$

$$u = Ax^2y,\ v = By^2zt,\ w = Czt^2$$

The acceleration $f$ of the fluid Particle is Given as

$$f = \frac{\partial q}{\partial t} + u\frac{\partial q}{\partial x} + v\frac{\partial q}{\partial y} + w\frac{\partial q}{\partial z}$$

$$f = jBy^2z + k2Czt + Ax^2y(i2Axy) + By^2zt(iAx^2 + j2Byzt) + Czt^2(jBy^2t + kCt^2)$$

$$\Rightarrow f = A(2Ax^3y^2 + Bx^2y^2zt)i + B(y^2z + 2By^3z^2t^2)j + C(2zt + Czt^4)$$

Ans.

which determine the acceleration of a fluid Particle.

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